



Rewarding Learning

General Certificate of Secondary Education

Further Mathematics

Unit 3
Statistics

[GFM31]

Assessment

MARK SCHEME

GCSE MATHEMATICS

General Marking Instructions

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

Assessment Objectives

Below are the assessment objectives for GCSE Further Mathematics.

Use and apply standard techniques (AO1)

Candidates should be able to:

- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

Reason, interpret and communicate mathematically (AO2)

Candidates should be able to:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

Solve problems within mathematics and in other contexts (AO3)

Candidates should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;
- evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

Additional guidance for teachers

These notes explain how the marks allocated in the published mark scheme are to be applied.

In the mark scheme, M indicates method marks and W indicates work marks. MW indicates a combined method and work mark. Work marks should not be awarded if the method is incorrect.

If a candidate misreads a question, eg copies a given equation incorrectly, deduct 1 mark and then FT (follow through), as long as the question is not made easier. However, work marks should not be awarded to answers which are inconsistent with the question, eg negative numbers of people.

If a correct answer in working is transferred incorrectly (or not at all) to the answer line, give BOD (benefit of doubt). However, if the incorrect answer is used in a subsequent section, penalise one mark in the subsequent section and treat the remaining work as a misread.

As a general rule, ignore work that is scored out.

However, if correct working is scored out, but subsequently used correctly, give BOD.

If incorrect working is not scored out but subsequently corrected, give BOD.

If more than one attempt at a question is made and none is scored out, mark the attempt that corresponds to the answer given in the answer line. If no answer is given in answer line, mark the worst attempt, unless the better attempt is clearly the one that is intended to be taken.

If incorrect or unnecessary working is given after a correct answer, ignore this if the correct answer is on the answer line. However, if a subsequent incorrect answer is given on the answer line deduct 1 mark.

Mark answers only in appropriate section. Allow forward marking if carried through into appropriate section, eg if working for part (ii) is given in part (i) and not repeated but used in part (ii), give appropriate marks in part (ii).

Do not allow backward marking, eg answer to part (ii) is given as answer to part (i) and no answer, or an incorrect answer, is given in part (ii).

All working must be shown, so answers with no working can get no marks, unless the working is trivial.

Answers should be exact where possible.

Unless specifically stated in the mark scheme, accept one or more dp (decimal places), or 3 significant figures, unless insufficient accuracy leads to very inaccurate or meaningless answer.

Ignore slight rounding errors, eg arising from calculations using values to 2 rather than 3 or more places. Accept 1.5 instead of 1.50 for an answer required to 2 dp. Ignore truncation error in the last digit unless a specific accuracy is required, eg 0.345 truncated to 0.34.

- 1 (i)** MW1 for correct answer for total marks for boys.
 MW1 for correct mean.
 Allow FT from incorrect total for boys if it is a simple arithmetical error following a correct method, to find the total marks for boys. Division by 9 must be correct.
- (ii)** MW1 for inserting correct values into expression.
 W1 for getting 84972.
 MW1 for expression for standard deviation for boys.
 Allow FT if 84972 is incorrect due to an arithmetical error, but method is correct, and also allow FT from an incorrect value for 85 from **(i)**.
 W1 for answer. Must be to at least 2 decimal places.
 Allow FT if previous MW1 has been awarded, but do not award if the values give the square root of a negative number.
- 2 (i) (a)** MW1 Must have a completely correct table. The top 1 may be omitted.
- (b)** M1 MW1 M1 for choosing correct line for expansion. Must have at least 5 terms correct.
 If triangle stops at 1 6 15 etc and $(p + q)^6$ is given, allow this M1.
 MW1 for a correct expansion of $(p + q)^7$.
- (ii) (a)** MW1 for using $p = 0.1$ and $q = 0.9$.
 M1 W1 M1 for using the correct term, $21p^2q^5$.
 W1 for a correct answer.
 Accept more than 3 decimal places or 1240029/10000000.
- (b)** M1 W1 M1 for choosing the correct terms.
 W1 for inserting the correct numerical values.
 W1 for a correct answer.
 No FT if previous M1 W1 has not been obtained.
 Accept more than 3 decimal places or 9743085/10000000.
- 3 (i)** M1 W1 M1 for correct values in expression for z .
 W1 for correct value for z .
 M1 for the correct expression. Diagram is not necessary.
 Accept the expression $1 - P(z < 1)$ or clear use of this expression.
 Allow FT for 1 if it was calculated incorrectly.
 W1 for answer. Can be to 4 decimal places.
 If previous M1 has been obtained, allow FT for 1, but answer must be between 0 and 1.
 If you don't see $P(z > 1)$ or $1 - P(z < 1)$, then an answer of 0.8413 will only gain 2 marks.
- (ii)** M1 W1 M1 for correct values in expression for z .
 W1 for correct value for z .
 M1 for the correct expression. Diagram is not necessary.
 Accept the expression $1 - P(z < 0.5)$ or clear use of this expression.
 Allow FT for -0.5 if it was calculated incorrectly.
 W1 for answer. Can be to 4 decimal places.
 If previous M1 has been obtained, allow FT for -0.5 , but answer must be between 0 and 1.
 If you don't see $P(z < -0.5)$ or $1 - P(z < 0.5)$, then an answer of 0.6915 will gain 2 marks.

- 4 (i) MW1 for a correct set of ranks in ascending or descending order.
 MW1 for a correct second set of ranks in the **same** order.
- (ii) No marks in this section if the ranks are in opposing order.
 M1 W1 M1 for evidence of calculating d^2 . All values must be positive and at least 3 values correct.
 W1 for getting $\Sigma d^2 = 30$.
 Allow FT from an error in ranks in (i).
 M1 for values in the expression for r .
 Allow FT for value for Σd^2 , as long as M1 for calculating d^2 has been obtained.
 W1 for r .
 Allow FT as long as previous M1 has been obtained and result gives $-1 < r < 1$.
- (iii) M1 for stating positive correlation, if $0 < r < 1$.
 Allow FT from (ii) – negative correlation if $-1 < r < 0$
 or ‘no correlation’ or ‘weak correlation’ if $-0.4 < r < 0.4$.
 Award no marks in this section if $||r|| > 1$.
- (iv) MW1 Both answers must be correct.
- (v) Allow FT from part (iv) as long as the means are in the range 68 – 72 and 70 – 74.
 M1 for a straight line with positive gradient through the means.
 Line must be within half a small square of the means point.
 Award this mark if the line goes through the means point, even if it is not plotted.
 W1 for the slope of the line.
 Accept any line which crosses the vertical axis between 48 and 60.
- (vi) M1 for valid method for calculating m .
 Must use 2 points on the line, preferably one being the means.
 Must be positive gradient to get this mark.
 M1 for valid method for calculating c , even if gradient is given as negative.
 This mark is independent of the previous M1.
 W1 for equation.
 Only award this mark if the two previous M1 marks have been obtained.
 As there can be a wide variation of acceptable lines on the graph, so there can be a wide variation of equations. However, the equation should be consistent with the line drawn.
- 5 (i) MW1 for having the 4 in the correct position.
 MW1 for having 1, 2 and 15 in the correct positions.
 No FT from an incorrect value for 4.
 MW1 for having 3, 7, 12 and 6 in the correct positions.
 Allow FT from any incorrect values for 1, 2 or 15, but no FT if 4 was incorrect.
- (ii) Correct answer with little or no working can get full marks.
 MW1 W1 MW1 for using (sum of three areas)/50.
 Allow FT from the diagram if 1, 2 and 15 are incorrect.
 W1 for answer. Must be between 0 and 1.

(iii) Correct answer with little or no working can get full marks.

MW1 for getting 18. Allow FT from diagram.

MW1 for getting 5. Allow FT from diagram.

MW1 for answer.

Allow FT but must have obtained previous two MW1 marks and answer must be between 0 and 1.

Alternative solution

M1 W1 W1 M1 for using the correct method.

W1 for using the correct values. Allow FT from diagram.

W1 for answer.

Allow FT, but answer must be between 0 and 1.

6 Correct answer with no correct equation gains no marks.

MW1 MW1 M1 MW1 for $x/(x+5)$

MW1 for $(x-1)/(x+4)$ **and** evidence of multiplying.

M1 for equation with product set equal to $1/6$.

MW1 for $5x^2 - 15x - 20 = 0$ **or** $x^2 - 3x - 4 = 0$

MW1 for factors or alternative method, e.g. using quadratic formula.

W1 for answer.

No FT from an incorrect quadratic equation.

Do not award mark if -1 is left in the answer.

If candidates form an equation **with** replacement, they can gain a maximum of 3 marks –

Give 2 marks for $x/(x+5) \times x/(x+5) = 1/6$

Give 1 mark for FT equation $5x^2 - 10x - 25 = 0$ **or** $x^2 - 2x - 5 = 0$.

If a candidate has this equation with the correct answer of $x = 4$, award no marks.

1 (i) Total marks for boys = $84 \times 12 - (92 + 80 + 71)$

$$= 765$$

MW1

Mean for boys = $\frac{765}{9} = 85\%$

MW1

(ii) For all marks

$$s^2 = \frac{\sum x^2}{12} - 84^2$$

MW1

$$\sum x^2 = 84972$$

W1

For boys

$$\sum x^2 = 84972 - (92^2 + 80^2 + 71^2) = 65067$$

St. dev. for boys = $\sqrt{\frac{65067}{9} - 85^2}$

MW1

$$= 2.16\%$$

W1

6

2 (i) (a)

				1				
			1		1			
			1	2	1			
		1	3	3	1			
	1	4	6	4	1			
	1	5	10	10	5	1		
	1	6	15	20	15	6	1	
1	7	21	35	35	21	7	1	

MW1

(b) $p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q^7$ M1 MW1

(ii) (a) $P(\text{green}) = p = 0.1$

$P(\text{white}) = q = 0.9$

$P(\text{exactly 2 green}) = 21p^2 q^5 = 21(0.1)^2(0.9)^5$
 $= 0.12$

MW1

M1

W1

(b) $P(\text{at most 2 green})$

$= q^7 + 7pq^6 + 21p^2 q^5$

$= (0.9)^7 + 7(0.1)(0.9)^6 + 21(0.1)^2(0.9)^5$

$= 0.97$

M1 W1

W1

9

3 (i) $z = \frac{75 - 65}{10} = 1$

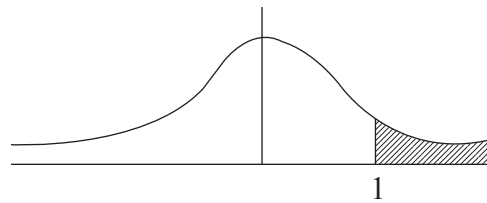
$P(\text{free TV licence})$

$= P(z > 1)$

$= 1 - P(z < 1)$

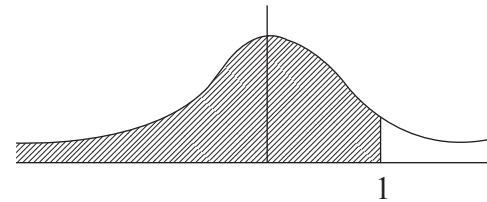
$= 1 - 0.8413$

$= 0.16$



M1 W1

M1



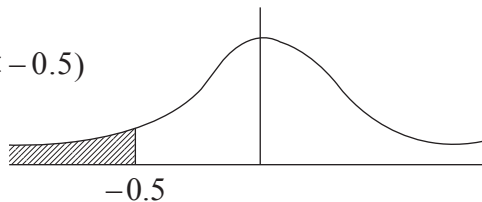
W1

(ii) $z = \frac{60 - 65}{10} = -0.5$

M1 W1

$P(\text{not travel pass}) = P(z < -0.5)$

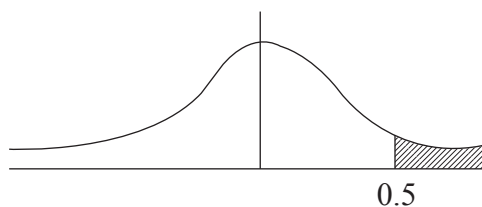
M1



$= 1 - P(z < 0.5)$

$= 1 - 0.6915$

$= 0.31$



W1

8

4 (i)

Ranks (Winter)	1	8	7	4	5	2	6	3	9	MW1
Ranks (Summer)	1	4.5	9	4.5	8	2.5	6	2.5	7	MW1

Alternatively

Ranks (Winter)	9	2	3	6	5	8	4	7	1	MW1
Ranks (Summer)	9	5.5	1	5.5	2	7.5	4	7.5	3	MW1

(ii)

d^2	0	12.25	4	0.25	9	0.25	0	0.25	4	M1 W1
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$$r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

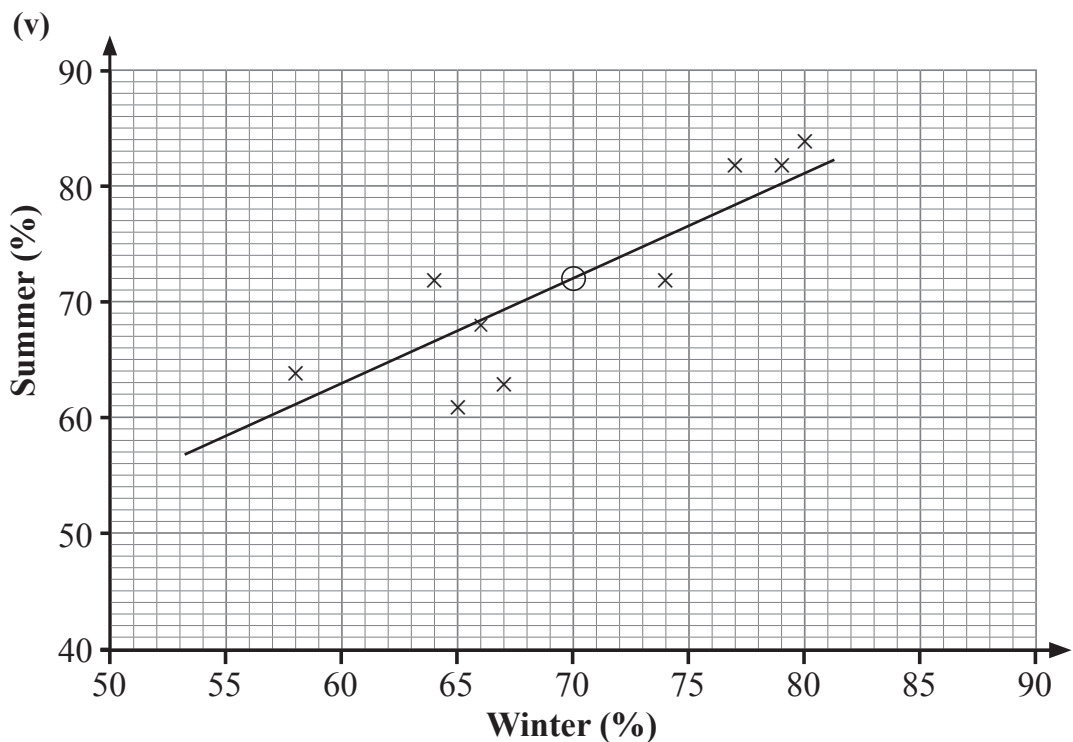
$$r = 1 - \frac{6(30)}{9(80)} \quad \text{M1}$$

$$r = 0.75 \quad \text{W1}$$

(iii) Positive correlation M1

(iv) Mean winter = $\frac{630}{9} = 70$

Mean summer = $\frac{648}{9} = 72$ MW1



M1 (through means)
W1 (slope)

AVAILABLE
MARKS

(vi) Gradient = $\frac{81 - 72}{80 - 70} = \frac{9}{10} = 0.9$

M1

Using means

$72 = 0.9(70) + c$

$c = 9$

M1

Equation is

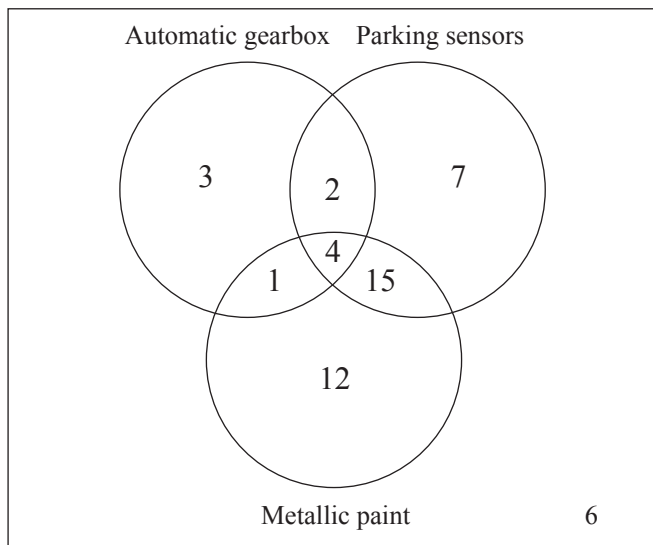
$y = 0.9x + 9$

W1

AVAILABLE
MARKS

13

5 (i)



(4) MW1

(1, 2, 15) MW1

(3, 7, 12, 6) MW1

(ii) No. with exactly 2 extras = $1 + 2 + 15 = 18$

$P(\text{exactly 2}) = \frac{18}{50}$ or $\frac{9}{25}$ or 0.36

MW1 W1

(iii) No. not choosing metallic paint = $50 - 32 = 18$

MW1

No. choosing auto gearbox but not metallic paint = $2 + 3 = 5$

MW1

$P(\text{auto gearbox} \mid \text{not metallic paint}) = \frac{5}{18}$ or 0.28

MW1

Alternative solution

$$P(\text{auto gearbox} \mid \text{not metallic paint}) \\ = \frac{P(\text{auto gearbox AND not metallic paint})}{P(\text{not metallic paint})}$$

$$= \frac{\cancel{5}/\cancel{50}}{\cancel{18}/\cancel{50}} = \frac{5}{18} \text{ or } 0.28$$

M1 W1 W1

AVAILABLE
MARKS

8

6 $\frac{x}{x+5} \times \frac{x-1}{x+4} = \frac{1}{6}$

MW1 MW1 M1

$$6x(x-1) = (x+5)(x+4)$$

$$6x^2 - 6x = x^2 + 9x + 20$$

$$5x^2 - 15x - 20 = 0$$

MW1

$$x^2 - 3x - 4 = 0$$

$$(x-4)(x+1) = 0$$

MW1

$$x = 4 \quad [x = -1 \text{ is impossible}]$$

W1

6

50